Study on the effect of statistics on the calculation of covariances and correlations with the GEF code

Karl-Heinz Schmidt, 30. 1. 2018

General description

Uncertainty analysis from calculations with perturbed parameters is available. These calculations are also used to determine covariances between different observables as required by the model. Covariances between the fission yields of two different systems can also be provided.

The variation of the perturbed parameters can be modified by a scaling parameter. Values smaller than one are useful for avoiding the problem that the yields of some nuclides become zero with some of the perturbed parameter sets, because GEF will not provide covariances/correlations for these nuclei. For example, a scaling factor of 0.5 may be used. Note that this leads to a reduction of the uncertainties by the same factor and a reduction of the variances and covariances by a factor of 0.25. The correlations are not systematically modified.

Another effect of the Monte-Carlo method that is used by the GEF code is a noise of fluctuations in the case of low statistics in the determined covariance/correlation values. In particular, covariances/correlations of strongly correlated quantities (correlation coefficients close to one) are reduced. This effect is particularly strong when covariances/correlations of independent yields of two different systems are considered. It is also aggravated when the variation range of the perturbed parameters is reduced as described above.

Both effects (lower yield threshold for calculated uncertainties/covariances/correlations and reduction of covariances/correlations of strongly correlated quantities) are reduced to the desired degree by performing the GEF calculation with higher statistics (a larger number of fission events).

Note that also the uncertainties determined by GEF include the effect of the statistical fluctuations in the case of insufficient statistics. In consequence, the calculated uncertainties are increased. This effect can be investigated by calculations with an increasing number of events. The influence of statistical fluctuations becomes negligible, when the uncertainties given by GEF attain an asymptotic value.

Illustrative examples

In the following, the influence of statistical fluctuations are shown for the matrix of correlations between the post-neutron mass yields of two identical systems $^{235}U(n_{th},f)$. Also the variation range of the perturbed parameters was modified. The cases are chosen to show the effects very clearly.

Figure 1 shows the correlations matrix, calculated with 10 000 events (enhancement factor = 1) and the default variation range of the perturbed parameters. The correlation pattern is clearly visible. However, there are two deficiencies: Firstly, some of the expected full correlation values of identical masses (on the diagonal line) are appreciably smaller than one. Secondly, there are gaps for masses near symmetry, where the correlation values are zero.

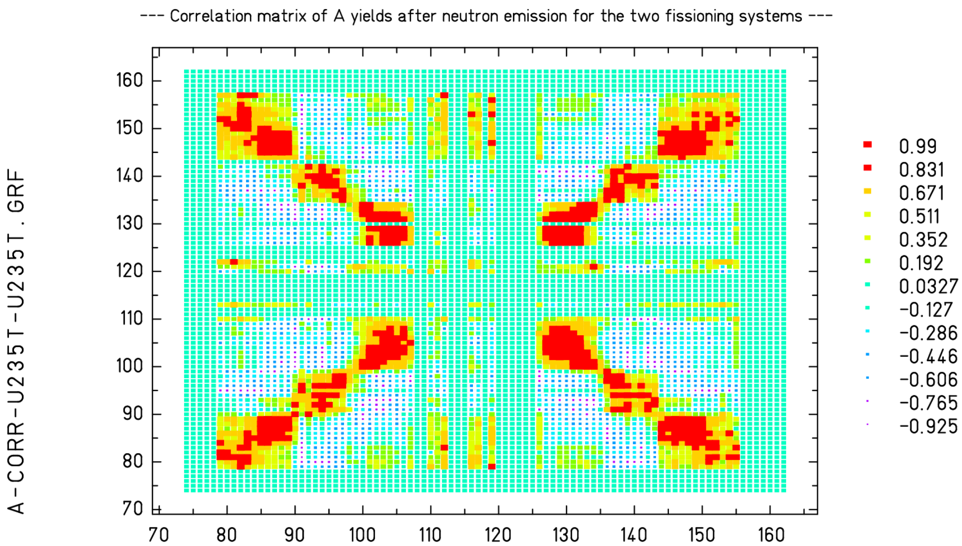

Figure 1: Correlation matrix of tthe mass yields of two systems $^{235}U(n_{th},f)$. GEF result with 10 000 events and default perturbation of parameters.

Figure 2 shows the correlation matrix, calculated with 10 000 events and a reduction of the variation range of the perturbed parameters by a factor of ½. The statistical noise has considerably increased, and the correlation pattern is blurred.

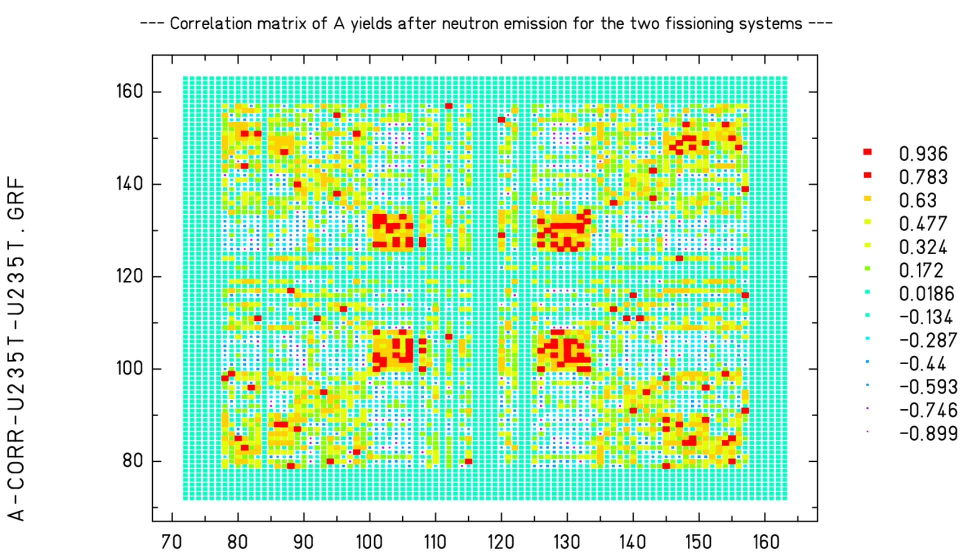

Figure 2: Correlation matrix of tthe mass yields of two systems $^{235}U(n_{th},f)$. GEF result with 10 000 events and perturbation of parameters reduced by a factor of 1/2.

Figure 3 shows the correlation matrix, calculated with 10 000 events and a reduction of the variation range of the perturbed parameters by a factor of 1/10. The statistical noise is even stronger, and the correlation pattern has almost disappeared.

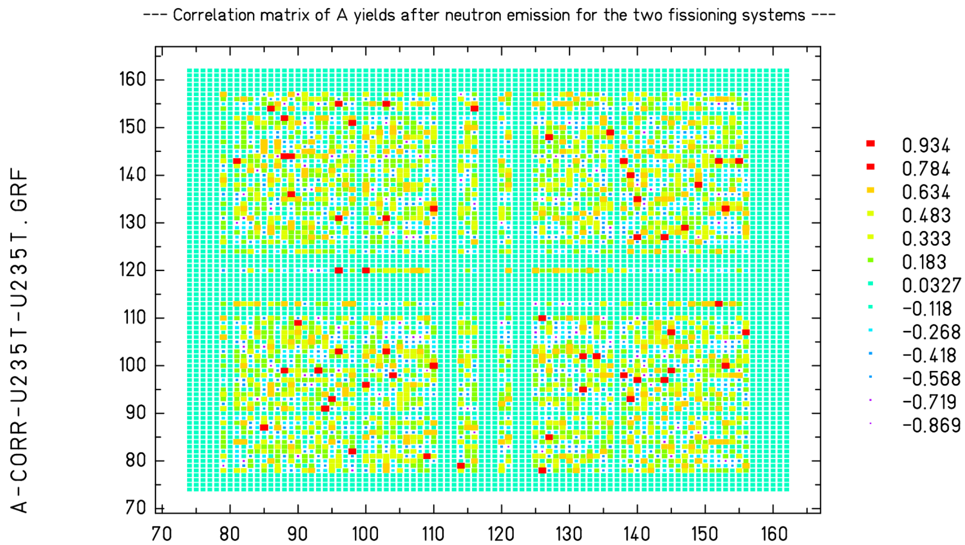

Figure 3: Correlation matrix of tthe mass yields of two systems $^{235}U(n_{th},f)$. GEF result with 10 000 events and perturbation of parameters reduced by a factor of 1/10.

Figure 4 shows the correlation matrix, calculated with 100 000 events and a reduction of the variation range of the perturbed parameters by a factor of ½. The correlation pattern is again clearly seen, and the gaps near symmetry are considerably filled up.

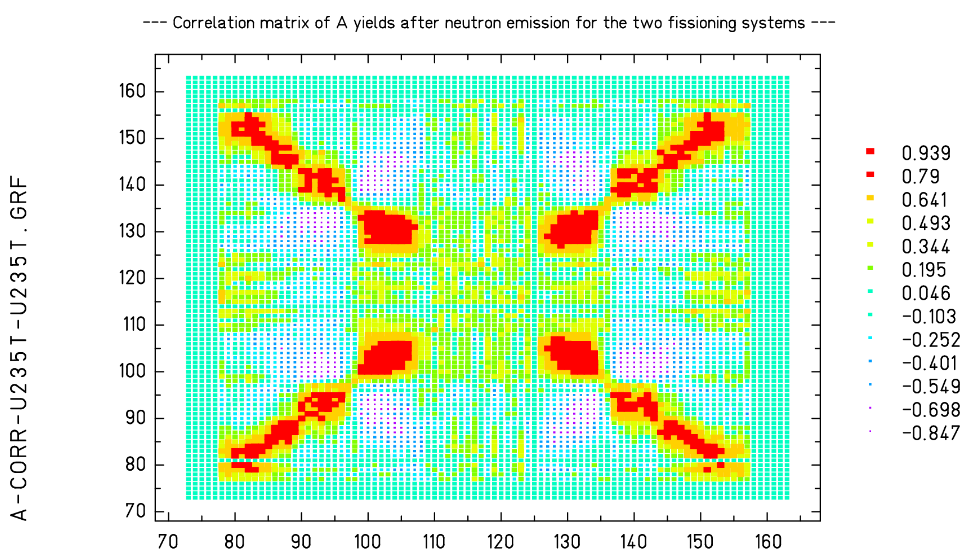

Figure 4: Correlation matrix of the mass yields of two systems $^{235}U(n_{th},f)$. GEF result with 100 000 events and perturbation of parameters reduced by a factor of ½.

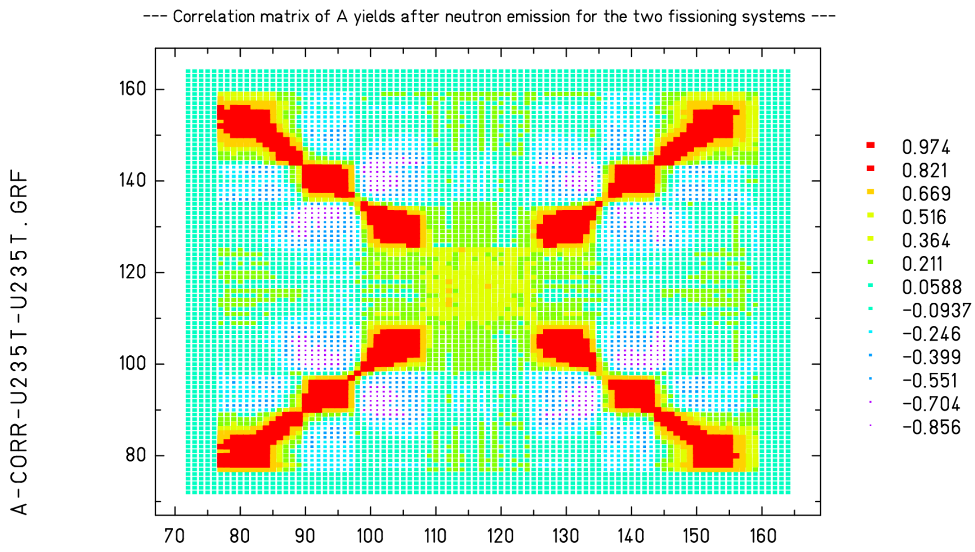

Figure 5: Correlation matrix of the mass yields of two systems $^{235}U(n_{th},f)$. GEF result with 1 000 000 events and perturbation of parameters reduced by a factor of ½.

Figure 5 shows the correlation matrix, calculated with 1 000 000 events and a reduction of the variation range of the perturbed parameters by a factor of ½. The correlation pattern appears even less disturbed, but the gaps near symmetry are not much changed compared to figure 4.

In order to obtain a even more undisturbed and full correlation matrix, the number of events should be increased by a factor of 10 or 100, and the variation range of the disturbed parameters may be slightly reduced.

From this study, one can extrapolate to the properties of the corresponding correlation matrix of the independent yields for the same case. If we assume that the yield of a certain mass is divided into about 4 independent yields with this mass value, the requirement on the statistics is correspondingly increased by a factor of 4 (or 10 to be on the safe side).

Summary

Two parameters are especially important for the quality of the correlation matrix from a GEF calculation: A reduction of the variation range of the perturbed parameters reduces the cut-off towards low yields. An increased number of calculated events reduces the statistical noise in the calculated correlation values. (These statements are also valid for the covariances and the uncertainties.)